Math Exploration Problems

1. A 5×5 grid is divided into five regions, as shown in the figure below. The empty cells are to be filled in by the digits 1, 2, 3, 4 or 5 such that each row, each column and each diagonal will have each digit once. The sums of all the digits in each region are equal. If two cells on the topmost row are filled by 1 and 2 (as shown), complete the grid by filling in the empty cells with the appropriate digits. What are the values of *A*, *B*, *C*, *D* and *E*?

			1	2
Α	В	С	D	Ε

2. The diagram below has nine circles connected by six line segments. One of the circles is already filled in by the integer 6. Choose nine consecutive positive integers (6 is included) and place each integer in an empty circle such that the sum of the integers in the circles located along each line segment equals 23.



3. Four teams *A*, *B*, *C*, *D* have a one week two-round robin game. Every two teams will play against each other twice. Each team can only play one game each day. The diagram below shows a part of the final scoreboard. The remaining part was split into four fragments (or pieces), with scores written on one side of each fragment (or piece). A black circle indicates winning of the team in the game, while the white circle indicates the lost of the team in the game. Show the arrangement of the fragments in the scoreboard. Which is the Champion Team?





4. The figure shown on the right can be formed using some or all the nets shown below. The distinct net can be rotated and reversed. Each net can only be used once. Which combination of distinct nets will form the figure shown? Use crayons to show the colored nets in the figure provided in the answer sheet.





- 5. A dartboard, divided into 20 sectors (or wedges), is shown below. Players take turns to throw 3 darts at it. Each dart scores a certain number of points, depending on where it hits the dartboard as follows:
 - 50 points for hitting the bull (The bull is the smallest circle at the center of the dartboard).
 - 25 points for a half-bull (The half-bull is the grey ring around the bull).
 - the number by a wedge, if it lands in the unshaded part of the wedge.
 - double the number by a wedge if it lands in the outer grey ring of the wedge.
 - triple the number by a wedge if it lands in the middle grey ring of the wedge.



Example: The three darts above score a total of $25 + (3 \times 13) + 12 = 76$.

Questions:

- a. Ruthee threw her three darts and scored 137 with two triples and a single (neither a bull nor half-bull). Find all the ways that she could have done this.
- b. On Archie's next turn, he scored 137 with two triples and a double. Find all the ways he could have done this.
- c. Archie threw three darts, all scoring a double or triple. What total scores could he *not* have got?

6. The arms of the weighing scale are divided into sections. A weight two sections away from the middle will be twice as heavy as a weight one section away. In which pan will you place the weights shown below in order to balance the whole weighing scale? Indicate your answer in the diagram by writing the weight (number of grams) in the corresponding pan.

